Mathematics for *Navigators*

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Devoted exclusively to mathematical preparation for the study of navigation, this book reviews addition, subtraction, [illegible] arithms, plane trigonometry, and spherical [illegible], for those who have previously studied these [illegible]

[illegible] taking up navigation without previous training [illegible] ches of mathematics, the book is sufficiently [illegible] e to enable them to approach navigation [illegible] gh understanding of the mathematics involved.

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CHAPTER IV

PLANE GEOMETRY

Definitions and Symbols. Nomenclature of the Triangle. Constructions. Nomenclature of the Circle. Using a Protractor. Exercise.

DEFINITIONS AND SYMBOLS

Geometry is that branch of mathematics which investigates the relations, properties, and measurement of lines, angles, surfaces, and solids. A brief summary of important facts in *plane geometry* is here given, in preparation for plane trigonometry, which follows.

In addition to those used in algebra, the following symbols are used in geometry:

$\triangle$ triangle	$\triangle ABC$	(*read* triangle ABC)
$\perp$ perpendicular	$AB \perp CD$	(*read* line AB is perpendicular to line CD)
$\parallel$ parallel	$AB \parallel CD$	(*read* line AB is parallel to line CD)
$\angle$ angle	$\angle A$ or $\angle CAB$	(*read* angle A or the angle CAB)

In navigation, the triangle is by far the most important plane figure. The nomenclature of angles follows:

An *angle* is formed by the intersection of two lines. If at a certain point there is but one angle, the angle is called by the name of that point. For example, in Fig. 8, the lines AC and BC intersect at C, and the angle at C is called simply "angle C." However, if there is more than one angle at a certain point, as in Fig. 9, it is necessary to refer to the angle between AC and BC as angle ACB. (The letter of the *vertex* of the angle is always placed in the

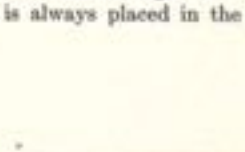

Fig. 8.

middle position.) In Fig. 9 there are three angles at C, namely, $\angle ACB$, $\angle BCD$, and $\angle ACD$.

If two *parallel* lines are cut by a third line (called a *transversal*), some of the resulting angles are equal, as illustrated in Fig. 10.

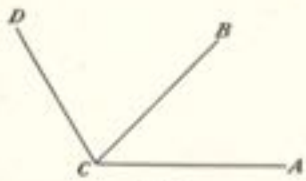

FIG. 9.

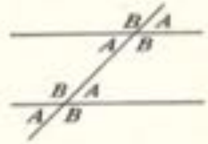

FIG. 10.

The angles with the same letter are equal. In addition, $A + B = 180°$.

A *right* angle is an angle formed by two lines that are perpendicular to one another. Figure 11 illustrates a right angle. A right angle measures 90°.

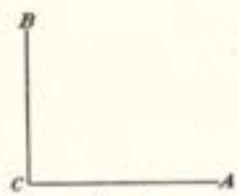

FIG. 11.

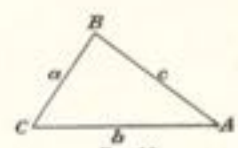

FIG. 12.

An *acute* angle is an angle *smaller* than a right angle. Figure 8 illustrates an acute angle. An acute angle measures *less* than 90°.

An *obtuse* angle is an angle *larger* than a right angle. In Fig. 9 $\angle ACD$ is an obtuse angle. An obtuse angle measures *more* than 90°.

NOMENCLATURE OF THE TRIANGLE

A plane triangle is a plane closed figure composed of three sides. Consequently, it has three angles. It is customary to letter the vertices of a triangle with the letters A, B, and C, and to label the sides opposite as a, b, and c, respectively, as shown in Fig. 12.

There are various kinds of triangles:

1. An *equilateral* triangle is one having three equal sides. An *equiangular* triangle is one having three equal angles. An equilateral triangle is equiangular, and vice versa. Figure 13 illustrates an equilateral and an equiangular triangle. In the figure, $\angle A = \angle B = \angle C$, and $a = b = c$.

2. An *isosceles* triangle is one having two sides equal. The angles opposite the equal sides are equal also. In Fig. 14, which illustrates an isosceles triangle, $\angle A = \angle C$, and $a = c$.

Fig. 13.

Fig. 14.

3. A *scalene* triangle is one having no sides equal, and consequently no angles equal. Figure 12 is a scalene triangle. In it, $a \neq b \neq c$, and $\angle A \neq \angle B \neq \angle C$. In any triangle, if two angles are unequal, the side opposite the larger angle is longer than the side opposite the smaller angle.

4. An *acute-angled* triangle (or *acute* triangle) is one having three acute angles. The triangle in Fig. 12 is an acute triangle. In it, each of the angles is smaller than a right angle. An acute triangle may be isosceles, scalene, or equilateral.

A *right-angled* triangle (or *right* triangle) is one having one right angle. Figure 15 illustrates a right triangle. A right triangle may be isosceles or scalene. The triangle in Fig. 15 is right and scalene. In a right triangle, it is customary to give the right angle the letter C, as in Fig. 15. The side c is called the *hypotenuse*. In a right triangle, $c^2 = a^2 + b^2$. The area of a right triangle equals $ab/2$.

An *obtuse-angled* triangle (or *obtuse* triangle) is one having an obtuse angle. Figure 16 illustrates an obtuse triangle. An obtuse triangle may be either isosceles or scalene. The triangle in Fig. 16 is obtuse and isosceles.

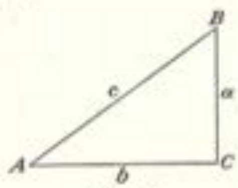

Fig. 15.

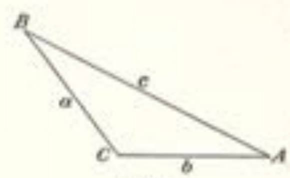

Fig. 16.

An *oblique* triangle is any triangle that is not a right triangle. An oblique triangle may be either acute or obtuse. Figures 12 to 14, and 16 are oblique triangles.

CONSTRUCTIONS

In plane geometry, certain elementary but important constructions may be performed with the aid of only a straightedge and compass. No scales for linear or angular measurement are

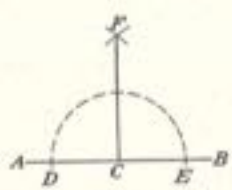

Fig. 17.

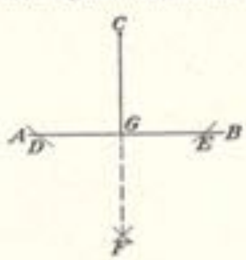

Fig. 18.

necessary. A few of these constructions will be described. Because of the brevity of this book, the proofs, which should be self-evident, are omitted.

To construct a perpendicular to a given line, at a given point in the line. In Fig. 17, the given line is AB, and the given point is C. With C as the center and any convenient radius, strike an arc cutting AB at D and E. With D and E as centers and any radius greater than DC, strike arcs intersecting above AB, at F. Draw FC, which is the required perpendicular to AB at C.

To construct a perpendicular to a given line, from a given point not on the given line. In Fig. 18, the given line is AB and the given point is C. With C as the center and any sufficient radius, strike an arc cutting AB at D and E. With D and E as centers, and any radius greater than half the distance DE, strike arcs intersecting below AB at F. Through C and F draw a line which meets AB at G. Then CG is the required perpendicular to AB

Fig. 19.

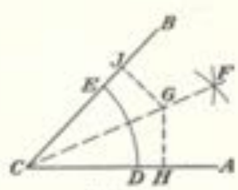

Fig. 20.

from C. The length of the perpendicular from a point to a line is the shortest distance between them and is referred to merely as *the distance* from the point to the line.

To construct the perpendicular bisector of a given line. In Fig. 19, the given line is AB. With A and B as centers and any convenient radius greater than half the length of AB, strike arcs intersecting above and below AB at C and D. Draw the line CD, which is the required perpendicular bisector of AB. Any point in the perpendicular bisector of a line is equally distant from the extremities of the line.

To construct the bisector of a given angle. In Fig. 20, the given angle is $\angle ACB$. With C as center and any convenient radius, strike an arc, cutting CA and CB at D and E, respectively. With D and E as centers and any radius greater than half the length DE, strike arcs intersecting at F. Draw the line CF, which is the required bisector of $\angle ACB$. Any point on the bisector of an angle is equally distant from the sides of the angle, or $GH = GJ$.

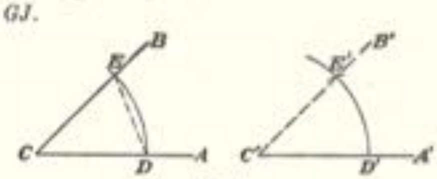

FIG. 21.

To construct an angle equal to a given angle. In Fig. 21, the given angle is $\angle ACB$. Draw any line, as $C'A'$ as one side of the required angle. With C and C' as centers and any convenient radius, strike arcs cutting CA, CB, and $C'A'$ at D, E, and D', respectively. With D' as center and DE as radius, strike an arc cutting the arc centered at C', at E'. Draw $C'B'$ through C' and E'. Then $A'C'B'$ is the required angle equal to $\angle ACB$.

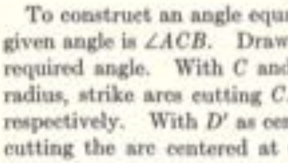

FIG. 22.

To construct a line parallel to a given line, through a given point (not on the given line). In Fig. 22, the given line is AB, and the given point is C. Draw a line from C to any point on AB, such as D, and extend CD to any point, as E. At C, construct $\angle ECF = \angle CDB$. Then, CF is the required line, parallel to AB, through C.

The sum of the three angles of a triangle is 180°. This statement can be proved as follows: Given, in Fig. 23, $\triangle ABC$. Extend side AC to some point, as E, and draw the line CD parallel to AB. The parallel lines AB and CD are cut by transversals BC and ACE. Therefore, $\angle B' = \angle B$, and $\angle A' = \angle A$. Now, it is obvious that $\angle C + \angle A' + \angle B' = 180°$. Therefore, $\angle C + \angle A + \angle B = 180°$. This proof also indicates the truth of the statement that "an external angle of a triangle ($\angle BCE$ in this case) equals the *sum* of the *remote* interior angles," or $\angle BCE = \angle A + \angle B$.

A median is a line drawn from a vertex of a triangle to the center of the opposite side. In Fig. 24, if $CD = DA$, then BD is the median from B. The three medians of a triangle meet in one point, which is the center of gravity of the triangle. Any line from CB to BA, parallel to CA, is bisected by BD.

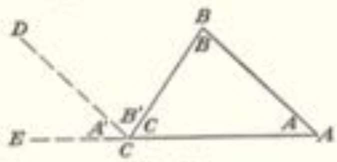

FIG. 23.

An altitude of a triangle is a perpendicular from a vertex to the opposite side. In Fig. 25, if $BD \perp AC$, then BD is an altitude of the triangle. Referring to Fig. 16, it will be seen that in the case of an obtuse triangle, the sides b and a must be extended to receive the altitudes from B and A. The area of any triangle is half the product of any side and the altitude to that side.

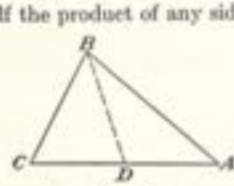

FIG. 24.

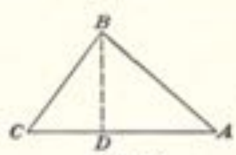

FIG. 25.

Since the perpendicular bisector of a line is everywhere equidistant from the extremities of the line, it follows that the perpendicular bisectors of the sides of a triangle must meet in a point, and that with this point as center, a circle may be drawn which will pass through the three vertices of the triangle. This circle is called the *circumscribed* circle. Figure 26 illustrates the intersection of the perpendicular bisectors, and the circumscribed circle.

Since the bisector of an angle is everywhere equidistant from the two sides of the angle, it follows that the bisectors of the angles of a triangle meet in a point which is equally distant from the three sides of the triangle. This point is called the *center* of the inscribed circle, which is a circle that just touches the three sides of the triangle. In Fig. 27, the bisectors of the angles are shown meeting in a point. The inscribed circle is also illustrated.

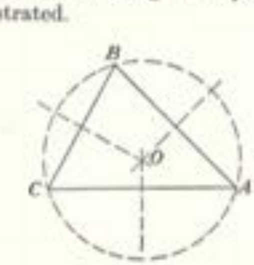

FIG. 26.

FIG. 27.

If the three angles of one triangle are respectively equal to the three angles of another triangle, the triangles are *similar*. In Fig. 28, $\angle A$, B, and C, are equal, respectively, to $\angle A'$, B', and C'. Therefore the $\triangle ABC$ and $A'B'C'$ are similar. In similar

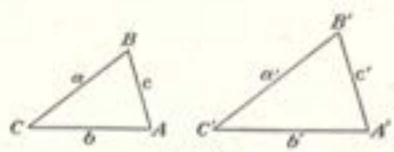

FIG. 28.

triangles the sides are proportional, that is, $a/a' = b/b' = c/c'$, or $a/b = a'/b'$, etc.

NOMENCLATURE OF THE CIRCLE

A *circle* is a plane closed figure, every point on the boundary of which (called the *circumference*) is the same distance from a point within, called the *center*. The various elements and divisions of a circle are as follows (refer to Fig. 29):

An *arc* is a portion of the circumference between two points. FA and AEB are arcs. An arc is measured by the angle at the center of the circle between the radii drawn to the extremities of the arc. Arc FA is measured by $\angle FOA$.

A *chord* is a straight line joining any two points on the circumference of a circle. AB and BC are chords.

A *diameter* is a chord that passes through the center of the circle. AOC is a diameter.

A *radius* is a straight line from the center to any point on the circumference. OF, OA, and OC are radii.

A *tangent* is an external straight line that touches the circumference at only one point, no matter how far it may be prolonged. HCG is a tangent. A tangent is perpendicular to the radius drawn to the point of tangency. $OC \perp HCG$ at C.

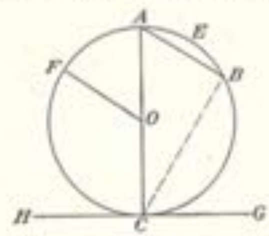

Fig. 29.

A *segment* is an area bounded by a chord and its subtended arc. $AEBA$ is a segment.

A *sector* is an area bounded by two radii and the subtended arc. $AOFA$ is a sector.

The angle between two chords that meet on the circumference is measured by half the arc intercepted between its sides. $\angle ACB = \frac{1}{2}$ arc AEB. Likewise, the angle between a chord and a tangent is measured by half the intercepted arc, or $\angle BCG = \frac{1}{2}$ arc BC.

The ratio between the length of the circumference of a circle and its diameter is about 3.1416, or $\frac{22}{7}$, and is represented by the symbol π (pronounced "pie"). Therefore, the circumference equals $\pi \times$ diameter, or $2\pi \times$ radius. The area of a circle is equal to $\pi \times (\text{radius})^2$, or πr^2.

USING A PROTRACTOR

A protractor is a device employed in plane geometry to measure angles. It may be used to construct an angle whose value is given or to measure a given angle whose value is desired. These

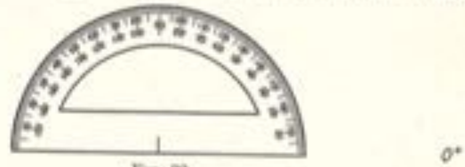

FIG. 30.

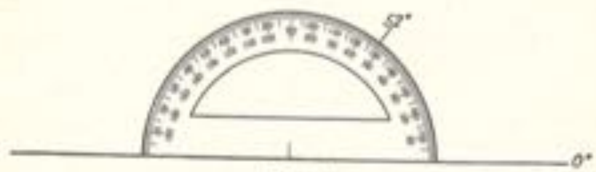

FIG. 31.

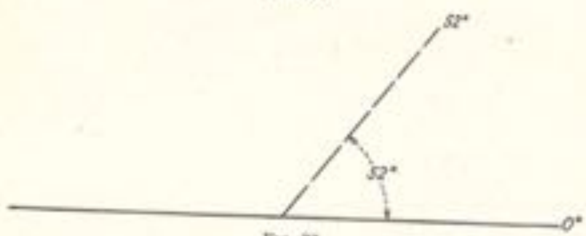

FIG. 32.

two uses will be described briefly. A protractor is illustrated in Fig. 30.

1. To construct an angle of a given size:

For example, let us construct an angle of 52°. First, draw a line of any convenient length. Place the center of the protractor at any point on the line. Mark this point and align the *base line* of the protractor with the given line. Then measure off the given angle (52° in this case) around the circumference of the protractor and mark the point. Remove the protractor and draw a line between the point on the line and the other point. This procedure is illustrated in Figs. 31 and 32.

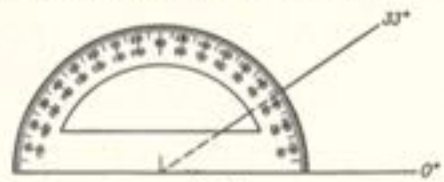

Fig. 33.

2. To measure a given angle:

Lay the protractor over the angle with the center of the protractor at the vertex and the base line of the protractor along one side of the angle. Note where the other side of the angle (prolonged if necessary) cuts the circumference of the protractor and read the angle. This is illustrated in Fig. 33, in which the angle is 33°.

EXERCISE

The student should actually make all the constructions described in this chapter and practice laying off angles of various sizes and reading given angles with the protractor.